**Bunbury Senior High School Name:\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_**

Test 5 Total Marks \_\_\_\_\_\_\_ / 50 \_\_\_\_\_\_\_\_%

**Mathematics: Methods Unit 2**

**Test 5, 2018**

**Arithmetic and Geometric Sequences and Series**

**\_\_\_\_\_\_\_ / 21 Marks**

**23 Minutes**

**Calculator Free**

**No Calculators Allowed**

**SCSA Formula Sheet Permissible**

**Arithmetic sequences**

* + 1. recognise and use the recursive definition of an arithmetic sequence:
    2. develop and use the formula for the general term of an arithmetic sequence and recognise its linear nature
    3. use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest
    4. establish and use the formula for the sum of the first terms of an arithmetic sequence

**Geometric sequences**

* + 1. recognise and use the recursive definition of a geometric sequence:
    2. develop and use the formula for the general term of a geometric sequence and recognise its exponential nature
    3. understand the limiting behaviour as of the terms in a geometric sequence and its dependence on the value of the common ratio
    4. establish and use the formula for the sum of the first terms of a geometric sequence
    5. use geometric sequences in contexts involving geometric growth or decay, such as compound interest

**Question 1 [2, 2, 3 =7 marks]**

The following sequences are either arithmetic or geometric. Determine the explicit and recursive formula for each sequence.

a) 200, 100, 50, 25…

Explicit 🗸

Recursive Tn+1 = (0.5)Tn T1 = 200 ✓

b) 16, 24, 32, 40, …

Explicit

🗸

🗸

Recursive T1 = 16 ✓

c) 625, -125, 25, -5, ……

Explicit =625 🗸

Recursive Tn+1 = −0.2×Tn T1 = 625 🗸

Includes T1 in all recursive formulas 🗸 -1 if not included.

**Question 2 [1, 1, 1 = 3 marks]**

A sequence starting with when graphed is linear.

1. What type of sequence is it?

Arithmetic

1. What does the gradient represent in the sequence?

The gradient represents d, the common difference in the sequence. 🗸

1. What does the vertical intercept represent in the sequence?

The difference between the first term and the difference pattern 🗸

**Question 3 [1, 2, 2 = 5 marks]**

For the arithmetic sequence with first term *a* = 100 and common difference *d* = -2 determine,

a) the first 4 terms in the sequence.

100, 98, 96, 94 ✓

b) T20

T20 = 100 + (20 – 1)×-2 ✓

= **62** ✓

c) S20

S20 = ✓

= 10(200 ) = **1620**  ✓

OR

S20 = 🗸

=1620 🗸

**Question 4 [2, 1, 1 = 4 marks]**

A geometric sequence has T4 = 16 and T9 = .

a) Determine the common ratio, *r* , of the sequence.

=

= 🗸

🗸

b) Determine the value of the first term T1 = *a*  of the sequence.

T4 =

16 = *a*

16 = *a* or any other method.egT1, T2, T3, T4

128, 64, 32, 16

***a* = 128**  ✓

c) Find the value of T6.

T6 = 128 T4, T5, T6,

16, 8, 4

= 4 ✓

**Question 5 [2 marks]**

Analyse the following geometric series. Which ones would have a S∞? Circle the correct response(s).

I A geometric series with first term *a* = 500 and common ratio = −2

II A geometric series with first term *a* = 400 and common ratio *r* = 0.75

III A geometric series with first term *a* = -5 and common ratio *r* = 4

IV A geometric series with first term *a* = 100 and common ratio *r* = −0.62

**Bunbury Senior High School Name:\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_**

**Mathematics: Methods Unit 2**

**Test 5, 2018**

**Arithmetic and Geometric Sequences and Series**

**\_\_\_\_\_\_\_ / 29 Marks**

**33 Minutes**

**Calculator ASSUMED**

**CAS Calculators Allowed**

**1 A4 Page of Written Notes Allowed**

**SCSA Formula Sheet Permissible**

**Arithmetic sequences**

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    2. develop and use the formula for the general term of an arithmetic sequence and recognise its linear nature
    3. use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest
    4. establish and use the formula for the sum of the first terms of an arithmetic sequence

**Geometric sequences**

* + 1. recognise and use the recursive definition of a geometric sequence:
    2. develop and use the formula for the general term of a geometric sequence and recognise its exponential nature
    3. understand the limiting behaviour as of the terms in a geometric sequence and its dependence on the value of the common ratio
    4. establish and use the formula for the sum of the first terms of a geometric sequence
    5. use geometric sequences in contexts involving geometric growth or decay, such as compound interest

**Question 6 [3, 3 = 6 marks]**

16000 tonnes of lithium were mined at the TALISON Greenbushes mine in each of the first two years of the mine’s operation. In the third year the quantity mined was 95% of the second years output, the fourth was 95% of the third year and so on. The mine will have to close at the end of the first year that the amount mined falls below 2000 tonnes.

1. For how many years will the mine remain open?

16000, 16000,

Solve 🗸

n =40.54 🗸

Falls below 2000 in the 43rd year. ( 41 years plus the first two years.) 🗸

1. What total tonnage of lithium was mined from this mine? Give your answer to the nearest 100 tonnes.

🗸

= 280932.3044 🗸

Total tonnage = 32000+ 280900 =312900 tonnes 🗸

**Question 7 [3, 2, 2 = 7 marks]**

A geometric sequence has T7 = 28672 and T9 = 458752.

a) Show that the common ratio of the sequence could be one of two possible values

and determine these values.

*r* 2 = 16 ✓

*r* = **4** or *r* = **−4**

* ✓

b) Determine the value of the first term T1 = *a* of the sequence.

T7 = 28672= *a* × ✓ Note:

✓

c) Is it possible to determine the infinite sum for this sequence? Justify your decision.

NO

Since the common ratio is not an infinite sum is not possible. 🗸

Justification essential for the mark.

**Question 8 [2, 1, 2 = 5 marks]**

A patient’s body absorbs a drug in such a way that whatever is in the body at a particular time, 40% remains in the body 24 hours later. Every 24 hours for the rest of the patient’s life an injection containing 30 mg of the drug must be taken.

In the long term how many mg of the drug will be in the patient’s body,

1. immediately after each injection?:

a=30 and r= 0.4 🗸

**🗸**

1. immediately before each injection?

🗸

1. The patient requires a more stable level of medication in her system and an alternative drug is found where 50% remains in the body 24 hours later. If in the long term 80 mg of the drug is required to be in the body immediately after each injection, how much drug is required in each daily injection?

**🗸**

🗸

40mg of the drug in the daily injection is required

**Question 9 [1, 2, 3 = 6 marks]**

When a particular ball is dropped onto a horizontal surface the height it reaches on its first bounce is 70% of the height of the height of the previous bounce. Subsequent bounce heights are 70% of the previous bounce. If the ball is dropped from a height of 3 metres, onto a horizontal surface, find the following, giving each answer correct to the nearest centimetre.

1. the maximum height reached after the first bounce

3,

Height reached on the first bounce is 2.10cm 🗸

1. the maximum height reached after the 5th bounce.

🗸

Height reached is 0.50cm🗸

1. the total vertical distance the ball travels before coming to rest.

2 recognises need for 2🗸

🗸

Total distance travelled is 17.00m🗸 recognises need to add 3m

**Question 10 [ 3, 3 = 6 marks]**

A property is purchased for $750000 in 2018. The real estate agent claims it is a great deal as the property will appreciate by 4% per annum. If the investor plans to keep the property for ten years and then consider selling it,

1. find the value of the property after this time. (to the nearest $100)

🗸

🗸

🗸

1. in what year will the property have doubled in value?

🗸

🗸

It will take 18 years for the property to double in value.

In 2036 the property will have doubled in value.🗸